Université d'Aix-Marseille | Master CMB-Bio
2020-2021

Continuous dynamical systems and modeling

Florence HUBERT - Charlotte PERRIN

Exercise 1. *Carbon-14 dating*
The carbon contained in living matter consists essentially in the stable isotope C^{12} (6 neutrons and 6 protons), but a tiny part is represented the radio-active isotope C^{14} (8 neutrons and 6 protons). This radio-active component comes from the cosmic radiation in the upper atmosphere. Thanks to a complex exchange process, the living matter maintains a constant portion of C^{14} in its total mass of carbon.
After death, the exchanges cease and the quantity of radio-active carbon decreases: it looses 1/8000 of its mass every year. This process enables scientists to date the death of the individuals from the analysis of the bones.

(1) Determine the differential equation satisfied by the mass of carbon 14 contained in a "dead" bone.
(2) Represent the shape of the solution curves for different initial conditions.
(3) Applications:
 (a) Skeletons from Neanderthal humans have been found in Palestine. Analysis show that to portion of C^14 is about $6,24\%$ of the mass of C^14 contained in a living body. Estimate the date of the skeleton.
 (b) Conversely, one estimates that Cro-Magnon man was living in France in the period between 30000 and 20000 BC. Determine an interval for the ratio between the mass of C^{14} in the skeleton and the one in a living body.

Exercise 2. We consider a specie whose population (i.e. the number of individuals) has doubled in 100 years and tripled in 200 years. Show that this population cannot follow the Mathusian dynamics.

Exercise 3.
Psychologists have discovered that, regardless the study time, a person can learn at most a given number N^* of meaningless words. Furthermore, they have shown that the rate of learning is proportional to the number of words not yet learned. If we denote $N(t)$ the number of words learned at time t, the time evolution of N is then governed by the following differential equation:

$$N'(t) = k(N^* - N(t)).$$

We consider here $N^* = 100$, $k > 0$.

(1) Explain why, from the modeling point of view, it is reasonable to assume that k is positive.
(2) Determine the solution of the Cauchy problem associated to the initial condition $N(0) = 0$. Represent the time evolution of the solution for different values of the parameter $k > 0$.
(3) Assuming that a person learns 30 words in the first 5 minutes, determine the value of the parameter k.