Continuous dynamical systems and modeling
Linear ODEs - Stability

Exercise 1. For the following matrices

$$A = \begin{pmatrix} -3 & 2 \\ -1 & 0 \end{pmatrix} \qquad A = \begin{pmatrix} -1 & 1 \\ 1 & 0 \end{pmatrix}$$

$$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad A = \begin{pmatrix} -1 & 1 \\ -1 & 0 \end{pmatrix},$$

- find the general solution of the differential system $Y' = AY$;
- determine the long-time behavior of the solution and the nature of the equilibrium point $Y^* = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$;
- sketch the phase portrait.

Exercise 2.
We split a population into two categories: the child population whose size at time t is denoted $N_c(t)$ and the adult population whose size at time t is denoted $N_a(t)$. We assume that the time evolution of the two populations follows the coupled dynamics

$$\begin{pmatrix} N_c \\ N_a \end{pmatrix}' (t) = \begin{pmatrix} -k_a & k_b \\ k_a & -k_d \end{pmatrix} \begin{pmatrix} N_c \\ N_a \end{pmatrix} (t)$$

with $k_a = 1$, $k_b = \frac{1}{2}$, $k_d = 1$.

(1) Precise the biological meaning of each parameter k_a, k_b, k_d.

(2) Determine the general solution of this differential system.

(3) Prove that $(0,0)$ is the unique equilibrium point of the system. Determine the stability of this equilibrium point.

(4) Sketch the phase portrait in the neighborhood of $(0,0)$.