



## M2 internship + PhD project

# Modelling and mathematical analysis of complex gravity flows

### Scientific description:

Snow, rock or ice avalanches represent an important hazard in mountain regions. Climate change leads to an increase of the frequency of these events, especially of wet snow avalanches. The global warming might also generate rock avalanches by reduction in the cohesion of rock walls and in the stability of high-altitude glaciers. In order to assess the risks and implement appropriate protection measures, it is crucial to have robust mathematical models allowing numerical simulations in a reasonable computational time. From theoretical and modelling standpoints, models that have been proposed in the literature may be ill-posed (see [1]) and do not always take into account important physical properties like the heterogeneity of the medium. Besides, the rheology of the snow is still not well understood, especially for what concerns the transition between dry cold snow, behaving like a granular medium, and wet warm snow, behaving like a viscoplastic material. The main goals of this project are: the derivation and the analysis of depth-averaged granular models including new heterogeneity effects, such as compressibility, shearing and non-locality effects; and the coupling between granular and viscoplastic regimes.

The first part of the project will be dedicated to the derivation of compressible nonlocal granular models via Hamilton's principle, cf [2]. The idea is to obtain a second gradient model using variational tools, model which accounts for small scales (e.g. the size of the grains) in the macroscopic dynamics, but also for the maximal packing constraint. A mathematical analysis will be conducted in 1D.

The second part of the project will concern the derivation of depth-averaged models for compressible granular rheologies with a reference profile which accounts for the shear (see [4]). The model will be implemented in 1D for comparison and validation with experimental data of the team ETNA on granular flows and standing jumps [3].

The final work will consist in adding nonlocal effects in the depth-averaged models and address the issue of coupling the granular regime with a viscoplastic regime. For this last task, the project will benefit from the recent results obtained by the team ETNA on non-newtonian viscoplastic fluids (YIELDGAP project).

### References:

- [1] Barker, T., Schaeffer, D. G., Bohórquez, P., and Gray, J. Well-posed and ill-posed behaviour of the  $\mu(I)$ -rheology for granular flow. *Journal of Fluid Mechanics* 779 (2015), 794–818.
- [2] Burtea, C., Gavriluk, S., and Perrin, C. Hamilton's principle of stationary action in multiphase flow modeling. *Lecture Notes on HAL* < hal-03146159 >, 2021.
- [3] Faug, T., Childs, P., Wyburn, E., and Einav, I. Standing jumps in shallow granular flows down smooth inclines. *Physics of Fluids* 27, 7 (2015), 073304.
- [4] Richard, G. L., Rambaud, A., and Vila, J. P. Consistent equations for open-channel flows in the smooth turbulent regime with shearing effects. *Journal of Fluid Mechanics* 831 (2017), 289–329.

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**Where:** I2M Marseille for the internship and the first part of the PhD project; UR ETNA Grenoble for the second part of the PhD project.

**When/duration:** The Master internship will last four months starting from April or May 2022. It will be followed, if the internship is successful, by a three-year PhD contract starting September 1, 2022.

**Funding:** The PhD project has received a co-funding by the IMPT.

**Eligibility:**

Candidates must have a general education in mathematics at master level with skills in analysis of partial differential equations, numerical analysis and scientific computing. Candidates should be able to demonstrate motivation for interdisciplinary work and a strong eagerness to learn physics and mechanics.

**Application process:** Applications must include a cover letter, a CV, any document that might attest the academic results during the last two years, two recommendations letters and be sent directly to [charlotte.perrin@univ-amu.fr](mailto:charlotte.perrin@univ-amu.fr). The closing date is December 17 2021. After December 17 2021, please contact [charlotte.perrin@univ-amu.fr](mailto:charlotte.perrin@univ-amu.fr) or [gael.richard@inrae.fr](mailto:gael.richard@inrae.fr) to know if the position has already been granted or not.